Name of Participant : ____ (Statutory Name)

Index No.: ________ / ________ **Name of School** : ___________________________________

Date of Birth : ____ **(DD/MM/YY)**

6 April 2019

Hwa Chong Institution
Mathematics Learning And Research Centre

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Asia Pacific Mathematical Olympiad
For Primary Schools 2019

First Round
2 hours
(150 marks)

Instructions to Participants:

Attempt as many questions as you can.
Mathematical instruments, tables and calculators are not permitted.
Write your answers in the answer sheet provided.
Marks are awarded for correct answers only.

This question paper consists of 10 printed pages (including this page)

Number of correct answers for Q1 to Q10 : ______________ *Marks* ($\times$ 4) : ________________

Number of correct answers for Q11 to Q20 : ______________ *Marks* ($\times$ 5) : ________________

Number of correct answers for Q21 to Q30 : ______________ *Marks* ($\times$ 6) : ________________

1. If $\overline{ab} + \overline{ba} + b = \overline{aab}$, find $a + b$.

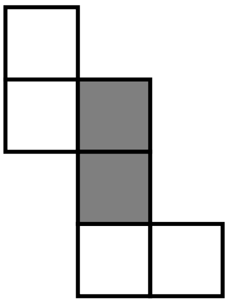
   ($\overline{ab}$ denotes a 2-digit number where the tens digit is a and the unit digit is b.)

2. Given a and b are positive integers (whole numbers excluding 0), how many ordered pairs of numbers (a, b) are there such that

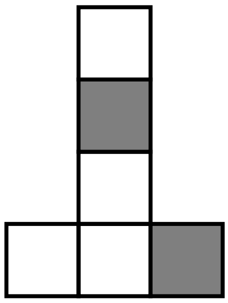
$$a^2 + b^2 \leq 14?$$

   For example, (1, 2) and (2, 1) are considered two different ordered pairs.

3. When these nets are folded to make cubes, which (if any) will have the two shaded faces directly opposite each other?



A



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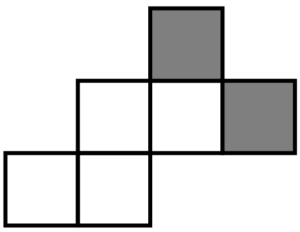

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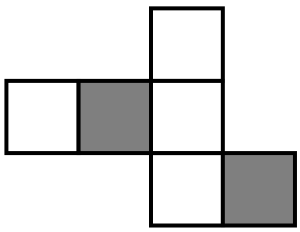
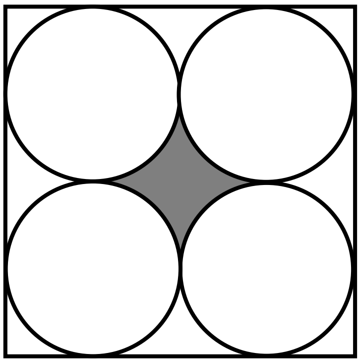


D

E. None of the given nets.

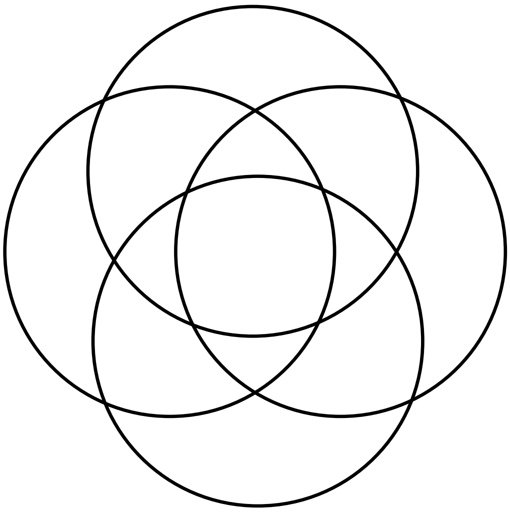
4. In the diagram below, four identical circles are drawn inside a square of area $1600\ \text{cm}^2$. Find the area of the shaded region. Take $\pi = 3.14$.



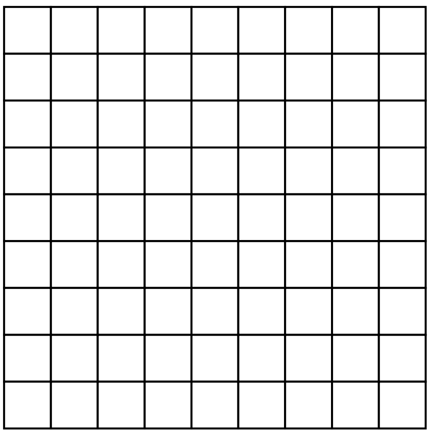
5. In a football tournament, there are *k* teams.
Each team plays against every other team exactly once.
Three points are awarded to a team for a win; two points for each of the two teams in a draw; one point for a loss.
At the end of the tournament, the total points of all the teams are 24.
Find the value of *k*.

6. Given
$$\frac{79}{137} = a - \cfrac{1}{b + \cfrac{1}{c - \cfrac{1}{d + \cfrac{1}{e}}}},$$
where *a, b, c, d, e* are whole numbers, find the value of $a + b + c + d + e$.

7. In the diagram below, each of the integers from 1 to 13 is filled in one of the regions created by the four circles. Each circle contains seven numbers. The sum of numbers in each circle is S_1, S_2, S_3, S_4.
   Find the largest value of $S_1 + S_2 + S_3 + S_4$



8. In a 9 × 9 square grid, exactly 29 of the unit squares are shaded.
   Other cells are white.
   All the rows and columns have at least one unit square shaded each.
   There are x columns and y rows with at least five shaded cells each.
   Find the largest value of $x + y$.



9. In a test, there are ten multiple choice questions.
   Four points are awarded for a correct answer.
   One point is deducted from the total for every wrong answer.
   No point is given for an unanswered question.
   How many different total points can the students score in this test?

10. Consider the table below where the numbers are arranged according to a pattern:

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	…	…	…	…

   The number 9 is in row 2 and column 4.
   If the number 2019 is in row *m* and column *n*, find the value of $m+n$.

11. A number *N* is divisible by each of the integers 2 , 3, 4, 5, 6, 8 and 9.
   N gives a remainder of 5 when divided by 7.
   Find the smallest value of *N*.

12. On a circular track of circumference 20 m, a robot A travels anticlockwise at a constant speed of 3.5 m/s, while another robot B travels clockwise at a constant speed of 1.5 m/s. They both start at the same point and at the same time.
   At most how many different points on the track will the two robots pass each other?

13. Find the sum of digits in the number

$$12345678910111213\ldots99989999.$$

14. A bag contains blue, white and red marbles.
The number of blue marbles is at least equal to half the number of white marbles, and at most equal to one third of the red marbles.
Given that the sum of the white marbles and the blue marbles is at least 55.
At least how many red marbles are there?

15. Arrange the three numbers below from the smallest to the largest:

$$x = \frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times \ldots \times \frac{47}{48}, \qquad y = \frac{2}{3} \times \frac{4}{5} \times \frac{5}{6} \times \ldots \times \frac{48}{49}, \qquad z = \frac{1}{7}.$$

16. Find the 2019$^{\text{th}}$ digit in the number

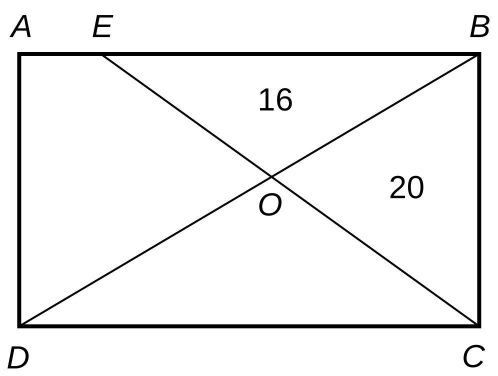
$$12345678910111213\ldots998999.$$

17. In how many ways can we choose two numbers from

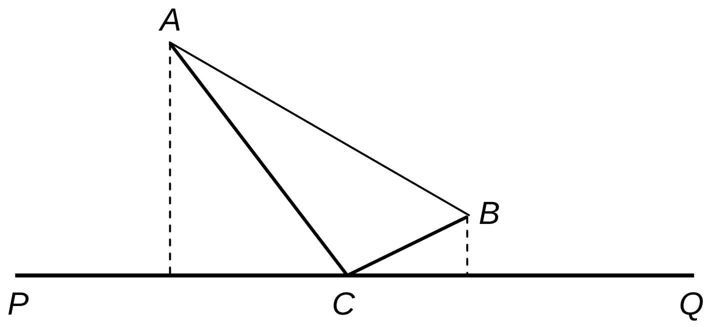
19, 20, 21, 22, …, 78, 79

such that the sum of the two numbers is even?

18. In a rectangle *ABCD*, the areas of two triangles are given.
If $AE = \frac{1}{5}AB$, find the area of quadrilateral *ADOE*.



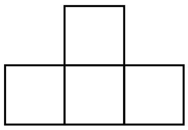
19. In the diagram below, $AB = 3$ m and $PQ = 8$ m. The perpendicular distances from *A, B* to the line *PQ* are 2 m and $\frac{1}{2}$ m respectively. A point *C* varies on the line *PQ*. Find the largest value of $AC - CB$.



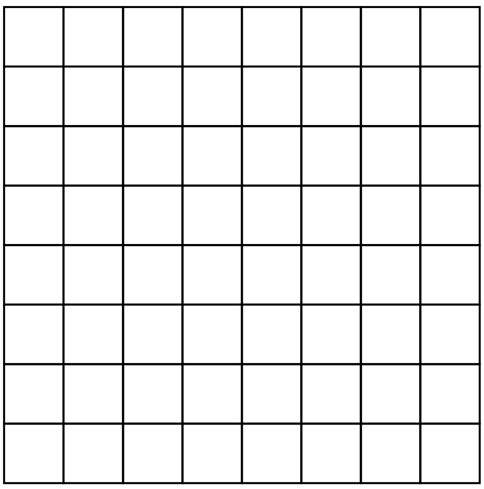
20. Among the integers: 1, 2, 3, …, 49, 50, what is the maximum number of integers that can be selected such that the sum of any two selected numbers is *not* divisible by 7?

21. Evaluate $\dfrac{(2^2+4^2+6^2+\cdots+100^2)-(1^2+3^2+\cdots+99^2)}{50}$.

22. Given an ordinary 8 by 8 square chessboard as shown, find the number of different ways of choosing one piece of



which is made up of four square units.



At time of print, only 22 questions are available.

Asia Pacific Mathematical Olympiad for Primary Schools 2019

First Round SOLUTIONS

Question 1: *(Answer: 10)*

$$\begin{array}{rrr} & a & b \\ & b & a \\ + & & b \\ \hline a & a & b \end{array} \quad \Rightarrow \quad \begin{array}{rrr} & 1 & b \\ & b & 1 \\ + & & b \\ \hline 1 & 1 & b \end{array}$$

Left hand side: $a + b$ + carry over = aa.

The carryover is at most 2. So, 9 + 8 + carryover < 20. Then, $a = 1$.

Then, b can only be 9 or 8. Testing the digits, only $b = 9$ works.

So, $a + b = 1 + 9 = 10$.

Question 2: *(Answer: 8 ordered pairs)*

$1^2 = 1, \quad 2^2 = 4, \quad 3^2 = 9, \quad 4^2 = 16 > 14$.

The ordered pairs are:

(1, 1), (1, 2), (1, 3),

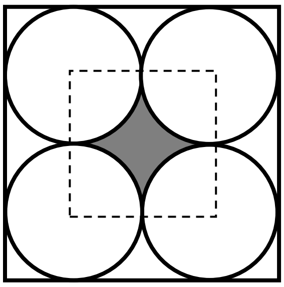
(2, 1), (2, 2), (2, 3),

(3, 1), (3, 2)

There are 8 possible ordered pairs.

Question 3: *(Answer: **D**)*

Question 4: *(Answer: 86* cm^2*)*



Length of Big Square = $\sqrt{1600} = 40$ cm.

Length of small square = (40 ÷ 4) × 2 = 20 cm.

Radius of a circle = 10 cm.

Area of shaded region

= Area of small square – Total area of 4 quadrants

= $20 \times 20 \;-\; 3.14 \times 10 \times 10$

= 400 – 314

= 86 cm^2

Question 5: *(Answer: k = 4)*

In every match, the total points obtained by both teams are always 4.

24 ÷ 4 = 6 matches in total.

6 = 1 + 2 + 3. So there are 4 teams: A, B, C, D. *k = 4.*

(The 6 matches are: A vs B, A vs C, A vs D; B vs C, B vs D; C vs D.)

Question 6: *(Answer: 15)*

$$\frac{79}{137}=1-\frac{58}{137}$$

$$=1-\frac{1}{\frac{137}{58}}=1-\frac{1}{2+\frac{21}{58}}$$

$$=1-\frac{1}{2+\frac{1}{\frac{58}{21}}}=1-\frac{1}{2+\frac{1}{\frac{(63-5)}{21}}}=1-\frac{1}{2+\frac{1}{3-\frac{5}{21}}}$$

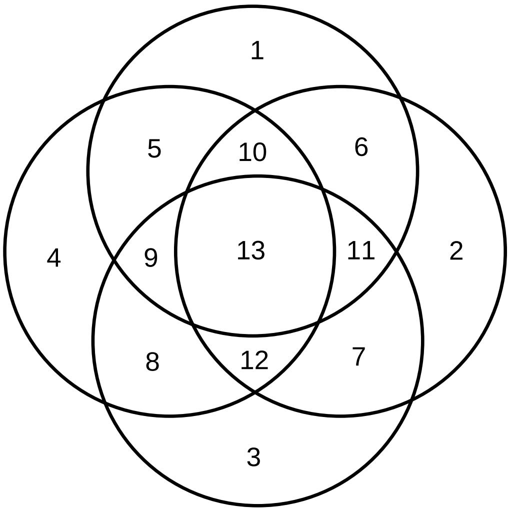
$$=1-\frac{1}{2+\frac{1}{3-\frac{1}{\frac{21}{5}}}}=1-\frac{1}{2+\frac{1}{3-\frac{1}{4+\frac{1}{5}}}}$$

$$a+b+c+d+e=1+2+3+4+5=15$$

<u>Question 7:</u> *(Answer: 240)*

To get the largest sum, place the larger numbers at regions of higher intersections.

An example is shown below.



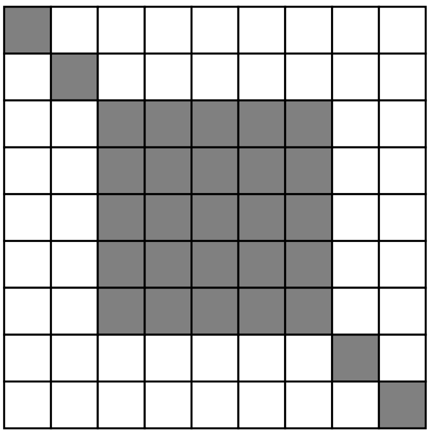
$$
\begin{aligned}
&S_1 + S_2 + S_3 + S_4 \\
&= \quad 1 + 2 + 3 + 4 \\
&\quad + 2 \times (5 + 6 + 7 + 8) \\
&\quad + 3 \times (9 + 10 + 11 + 12) \\
&\quad + 4 \times 13 \\
&= 10 + 52 + 126 + 52 \\
&= 240
\end{aligned}
$$

<u>Question 8:</u> *(Answer: 10)*

We need to shade 29 unit squares. First, shade the 9 diagonal unit squares black.

Next, shade the remaining 20 unit squares around the centre, giving at most 5 shaded unit squares to a row and column.

An example is shown below.



There are 5 rows and 5 columns with at least five cells shaded.

$x + y = 5 + 5 = 10$.

Question 9: *(Answer: 45 different total points)*

10 questions. One correct, 4 points; unanswered 0 points; each wrong, –1.

	Highest possible Score to Lowest possible Score
All 10 qts ✓	40,
9 qts ✓	36, 35,
8 qts ✓	32, 31, 30,
7 qts ✓	28, 27, 26, 25,
6 qts ✓	24, 23, 22, 21, 20,
5 qts ✓	*20*, 19, 18, 17, 16, 15,
4 qts ✓	*16*, *15*, 14, 13, 12, 11, 10,
3 qts ✓	*12*, *11*, *10*, 9, 8, 7, 6, 5,
2 qts ✓	8, 7, 6, 5, 4, 3, 2, 1, 0,
1 qts ✓	4, 3, 2, 1, 0, –1, –2, –3, –4, –5,
0 qts ✓	0, –1, –2, –3, –4, –5, –6, –7, –8, –9, –10.

$$(1+2+3+\cdots+11) - (1+2+3+\cdots+6)$$
$$=\frac{12\times 11}{2} - \frac{7\times 6}{2}$$
$$=66-21$$
$$=45$$

There are 45 different total scores possible.

Question 10: *(Answer: 256)*

$\frac{2019+1}{2}=1010$. So, 2019 is the 1010th odd number.

Note that in the table, the odd numbers are arranged in groups of eight:

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	…	…	…	…

$1010 \div 8 = 126\ R2$.

2019 is the 2nd number in the group after 126 groups of 8 odd numbers.

Row $m = 126 \times 2 + 1 = 252 + 1 = 253$.

Column $n = 3$.

$m + n = 256$.

Question 11: *(Answer: 1440)*

Lowest common multiple of: 2, 3, 4, 5, 6, 8, 9

$= 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$.

$N = 360$. $360 \div 7$ gives R3.
$N = 360 \times 2 = 720$. $720 \div 7$ gives R6.
$N = 360 \times 3 = 1080$. $1080 \div 7$ gives R2.
$N = 360 \times 4 = 1440$. $1440 \div 7$ gives R5.

The smallest possible value of *N* is 1440.

<u>Question 12:</u> *(Answer: 10)*

$20 \div (3.5 + 1.5) = 20 \div 5 = 4$ s.

The two robots will pass each other every 4 seconds.

In every 4 s, robot B travels $1.5 \times 4 = 6$ m.

On the 20 m track, they will pass each other at (m):

6, 12, 18, $24 \equiv 4$, 10, 16, $22 \equiv 2$, 8, 14, $20 \equiv 0$,

6, 12, 18, ….

The two robots will pass each other at 10 different points on the track.

<u>Question 13:</u> *(Answer: 180 000)*

Sum of Digits:

1 to 9: 45

$$\left.\begin{array}{l} 10, 11, 12, ..., 19; \\ 20, 21, 22, ..., 29; \\ \cdots \\ 90, 91, 92, ..., 99. \end{array}\right\} \begin{array}{l} \text{Sum of Digits in all 2-digit numbers:} \\ \text{(i) at One's place} = 45 \times 9; \\ \text{(ii) at Ten's place} = 45 \times 10. \\ \text{TOTAL} = 45 \times 19 \end{array}$$

(Hence, sum of digits in entire 1 and 2 digits numbers = 45×20.)

$$\left.\begin{array}{l} 100, 101, ..., 199; \\ 200, 201, ..., 299; \\ \cdots \\ 900, 901, ..., 999. \end{array}\right\} \begin{array}{l} \text{Sum of Digits in all 3-digit numbers:} \\ \text{(i) all 1 and 2 digit numbers} = (45 \times 20) \times 9 = 45 \times 180; \\ \text{(ii) at Hundred's place} = 45 \times 100. \\ \text{TOTAL} = 45 \times 280 \end{array}$$

(Hence, sum of digits in entire 1, 2 and 3 digits numbers

$= 45 \times 20 + 45 \times 280 = 45 \times 300$.)

$$\left.\begin{array}{l} 1000, 1001, ..., 1999; \\ 2000, 2001, ..., 2999; \\ \cdots \\ 9000, 9001, ..., 9999. \end{array}\right\} \begin{array}{l} \text{Sum of Digits in all 4-digit numbers:} \\ \text{(i) all 1, 2, 3 digit numbers} = (45 \times 300) \times 9 = 45 \times 2700; \\ \text{(ii) at Thousand's place} = 45 \times 1000. \\ \text{TOTAL} = 45 \times 3700 \end{array}$$

TOTAL SUM OF DIGITS FROM 1 TO 9999

$= 45 \times 3700 + 45 \times 300$

$= 45 \times 4000$

$=180\ 000$.

<u>Question: 14</u> *(Answer: 57)*

$W + B \geq 55$ and $B \geq \frac{1}{2}w$.

Hence, $W + \frac{1}{2}W \geq 55 \Rightarrow W \geq 55 \times \frac{2}{3} = 37$.

And, $B \geq \frac{1}{2}W \Rightarrow B \geq \frac{1}{2} \times 37 = 18\frac{1}{2} \Rightarrow B \geq 19$.

Now, $B \leq \frac{1}{3}R \Rightarrow 19 \leq \frac{1}{3}R \Rightarrow 57 \leq R$.

There are at least 57 red marbles.

<u>Question 15:</u> *(Answer: $x < y < z$)*

Multiplying *x* and *y*: $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \cdots \times \frac{47}{48} \times \frac{48}{49} = \frac{1}{49} < \frac{1}{7}$.

Hence, *z* is larger than *x* and *y*.

Since $\frac{1}{2} < \frac{2}{3}$, $\frac{3}{4} < \frac{4}{5}$, ..., $\frac{47}{48} < \frac{48}{49}$, *x* is smaller than *y*.

Thus, $x < y < z$.

<u>Question 16:</u> *(Answer: 9)*

1 to 9: 9 digits

$$\left.\begin{array}{l} 10,\ 11,\ 12,\ ...,\ 19; \\ 20,\ 21,\ 22,\ ...,\ 29; \\ \cdots \\ 90,\ 91,\ 92,\ ...,\ 99. \end{array}\right\}$$

There are 90 two-digit numbers.
Hence, $90 \times 2 = 180$ digits.

100 to 199: There are 100 three-digit numbers $\Rightarrow$ 300 digits.

Now, 2019 – 9 – 180 = 1830 (= 1800 + 30)

$$\left.\begin{array}{l} 100,\ 101,\ ...,\ 199; \\ 200,\ 201,\ ...,\ 299; \\ \cdots \\ 600,\ 601,\ ...,\ 699. \end{array}\right\} \ 1800 \div 300 = 6$$

So, 2019 is the 30th digit in 700, 701, …, 70***<u>9</u>***.

Hence, the 2019th digit is 9.

<u>Question 17:</u> *(Answer: 900)*

19,

20, 21, 22, 23, 24, 25, 26, 27, 28, 29,

….

70, 71, 72, 73, 74, 75, 76, 77, 78, 79.

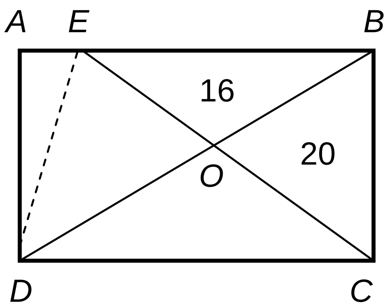
There are 30 even numbers and 31 odd numbers.

Choose both numbers as even: (30 × 29) ÷ 2 = 435.

Choose both numbers as odd: (31 × 30) ÷ 2 = 465.

Total: 435 + 465 = 900 ways.

Question 18: *(Answer: 29)*



Let $[\Delta BCO]$ denote the area of ΔBCO. So, $[\Delta BCO] = 20$.

Since $[\Delta CED] = [\Delta CBD]$, $[\Delta DEO] = 20$.

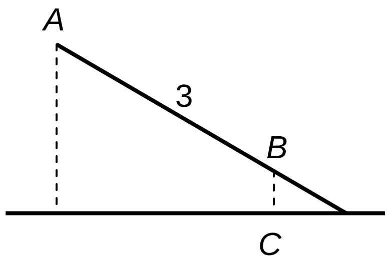
Now, $[\Delta BCE] = 16 + 20 = 36$ and $AE : EB = 1 : 4$. Hence, $[\Delta ADE] = 36 \div 4 = 9$.

Area of quadrilateral *ADOE* = 20 + 9 = 29.

Question 19: *(Answer: 3)*

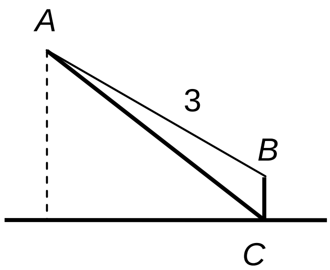
We just need to consider the following three cases:

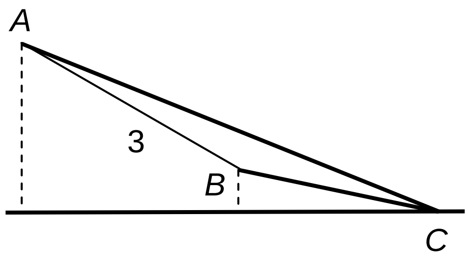
Case I:



$AC - CB = 3$

Case II and III:





In any triangle, the sum of any two sides is always larger than the third.

In triangle *ABC*, $3 + BC > AC \Rightarrow AC - BC < 3$.

Hence, largest value of $AC - CB$ is 3.

Question 20: *(Answer: 23)*

Arrange the numbers according to their remainders when divided by 7:

R0	R1	R2	R3	R4	R5	R6
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	32	33	34
35	36	37	38	39	40	41
42	43	44	45	46	47	48
49	50					

We choose numbers with R1, R2 and R3: 8 + 7 + 7 = 22 numbers.

No sum of two numbers is divisible by 7. So choose 1 number with R0.

Hence, at most 23 numbers.

Question 21: *(Answer: 101)*

Numerator: $2^2-1^2=3$, $4^2-3^2=7$, $6^2-5^2=11$, $8^2-7^2=15$, …..

In fact, $100^2-99^2=(100-99)\times(100+99)=199$.

Note: $(2n)^2-(2n-1)^2=4n^2-(4n^2-4n+1)=4n-1$.

The value of the numerator is:

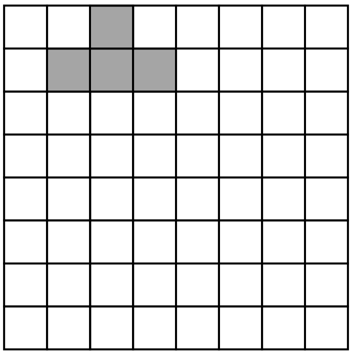
$$3+7+11+15+199$$
$$=\frac{(3+199)\left[(199+1)\div 4\right]}{2}$$
$$=\frac{202\times 50}{2}=101\times 50$$

Hence, the required value is: $\dfrac{101\times 50}{50}=101$.

<u>Question 22:</u> *(Answer: 168)*

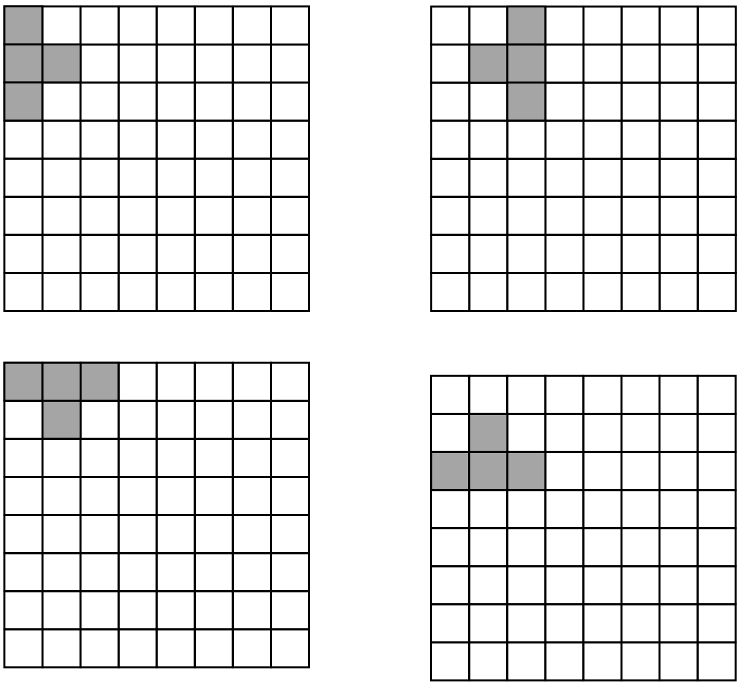
Along the edge of the board, excluding the four corners, for each cell, there is only 1 way to place the shape.

Hence, there are 6 × 4 = 24 ways.



Consider the inner 6 by 6 grid. At each cell, there are 4 ways to place the shape, as we can simply rotate it around.

A total of (6 × 6) × 4 = 144 ways.



GRAND TOTAL = 24 + 144 = 168 ways.